Measurement and Accuracy

Whenever you take a measurement, you are making an estimate. There is always an amount of uncertainty in measured values. In contrast, counted and defined values are exact numbers, and so have no uncertainty. For example, 32 students in a classroom is a counted number, and a length of 1 m is defined as exactly equal to 100 cm. There is no estimation in these values, and so no uncertainty.

Accuracy is the difference between a measurement and its true value. No matter how carefully you work, there will be a difference between a quantity you measure and its true value. The accuracy of any measurement is affected by the precision of the measurement. Precision refers to the degree of agreement among repeated measurements of the sample (the reproducibility). Precision is determined by your actions; how carefully you take measurements and control the variables in your experiment. Figure 5.1 illustrates the differences between precision and accuracy, using the example of a darts game.

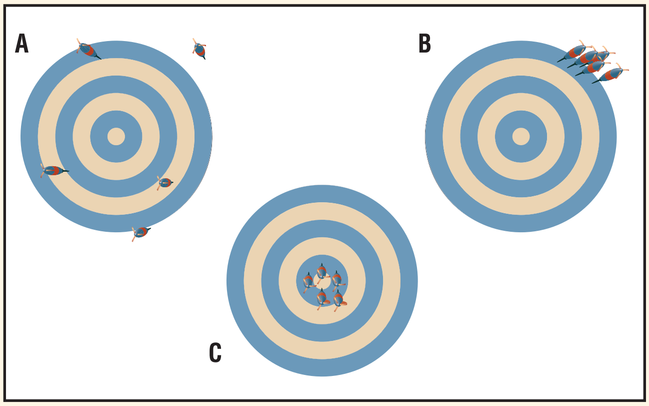


FIGURE 5.1 In this illustration, the centre of the dartboard is the true value of the measurement. Player A was neither precise nor accurate; the positions of the shots all differed and none hit the centre. Player B was precise but not accurate; all the darts hit the same area of the target, but they all were off the centre. Player C was both precise and accurate; all the darts are close to one another and in the centre of the target.

The accuracy of a measurement may be increased by carefully repeating it several times. When you repeat a measurement, you make another estimation of the true value of the quantity. Scientists often report an average (mean) of repeated measurements, since the average will usually be more accurate than only one measurement. However, it is possible to make reproducible measurements that are not accurate, as shown by Player B in Figure 5.1.

Choosing an appropriate instrument can also affect the accuracy of a measurement. All measuring instruments are limited by their readability and by their internal precision. Readability is the smallest fraction of a division on the scale of an instrument that can be read with ease, using estimation. Readability is affected by the size of the divisions on the scale, and is usually considered to be half of the smallest division.

For example, Figure 5.2 shows two rulers. Ruler A has only centimetre divisions, whereas ruler B has millimetre divisions. When ruler A is used to measure, you can see only that the length of the eraser is between the 2 cm and 3 cm marks. Using ruler A, you can therefore estimate the length to be 2.5 cm, but you know that the true value may be anywhere between 2.0 cm and 3.0 cm. If you use ruler B instead, you can see that the length of the eraser is between 2.3 cm and 2.4 cm. You can therefore estimate the length to be 2.35 cm using ruler B, and the true value of the measurement can be anywhere between 2.3 and 2.4 cm.

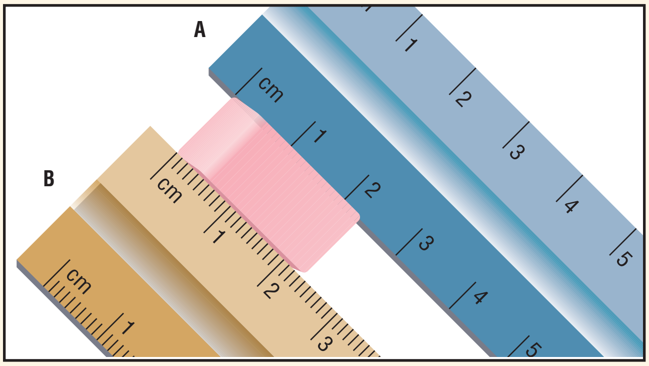


FIGURE 5.2 Which of these two rulers has a readability that would allow you to estimate a measurement of 2.35 cm for the length of the eraser?

Each measuring instrument has a certain internal precision. The precision of an instrument is the degree of agreement among repeated measurements of a single sample using that instrument. For example, a digital thermometer may be precise to within 0.2°C. This means if the thermometer reads 1.0°C, the true temperature of the substance could be anywhere between 0.8°C and 1.2°C.

Significant Digits

Significant digits are the specific number of digits used to communicate the degree of uncertainty in a measurement. The last digit indicates the uncertain (or estimated) digit. The measurement of 2.5 cm for the eraser taken with ruler A above has two significant digits, but the measurement of 2.35 cm taken using ruler B has three significant digits. When a measurement is on a division on a scale, indicate it by including a zero. For example, a length on the 3 cm mark would be recorded as 3.0 cm (two significant digits) on ruler A, and as 3.00 cm (three significant digits) on ruler B.

Student Reference 6: Math Skills

Solving problems related to science often requires you to apply math skills. In the inquiry process, math skills may be needed to analyze quantitative observations. In the problem-solving process, math skills may be needed to calculate scale or determine a budget. In the decision-making process, math skills may be needed to assess the arguments made for two sides of an issue.

Calculations with Significant Digits

Recall that significant digits are used to communicate the degree of uncertainty of a measurement. The last number in a measurement is always the digit that was estimated. When performing calculations with measured values, it is important that the uncertainty of the final result is also communicated correctly. This is done by using rules to count the number of significant digits in each number in the calculation, and to determine the correct number of significant digits in the final answer.

Rules for Counting Significant Digits

1. The *digits 1 to 9* are always significant. For example, all of the following examples have 3 significant digits: 321, 0.321, 0.000 032 1, 3.21×10^3, and 3.21.
2. The position of zeros in a number determines whether or not they count as significant digits.
   - *Leading zeros* are not significant. For example, both 0.321 and 0.003 21 have 3 significant digits.
   - All *trailing zeros* are assumed to be significant. For example, the following examples all have 3 significant digits: 500, 5.00, 0.500, and 5.00×10^3.
   - *Zeros positioned between the digits 1 to 9* are significant on either side of the decimal point. For example, 203 has 3 significant digits, 1.905 has 4 significant digits, and 100.746 has 6 significant digits.
3. For *logarithmic values*, such as pH, any digits to the left of the decimal are not significant. For example, a pH of 2.3 has only 1 significant digit, and a pH of 10 has no significant digits.
4. All digits in *exact numbers* are considered to have an infinite number of significant digits. These include counted values (7 oranges) and defined values, such as the 1 and 1000 in the statement "1 kg is 1000 g."

Rules for Performing Mathematical Operations

The number of significant digits in a calculated value can never be greater than the number of significant digits in the original data. In many cases, this requires that the final answer be rounded off to the correct number of significant digits. Note that calculators do not apply the guidelines for significant digits, so you must always determine the correct number of significant digits yourself after using a calculator.

- When the first digit to be dropped is less than or equal to 4, the preceding digit is not changed. For example, 7.4345 is rounded to 7.43, giving 3 significant digits.
- When the first digit to be dropped is greater than or equal to 5, the preceding digit is increased by one. For example, 7.4355 is rounded to 7.44, giving 3 significant digits.

1. When *adding* or *subtracting*, the calculated result has the same number of decimal places as the number with the least number of decimal places in the original data.

Example 6.1

Perform the following addition, and express the answer using the correct number of significant digits.

$$\begin{aligned} x &= 2.2\text{ kg} + 8.267\text{ kg} + 12.32\text{ kg} \\ &= 22.787\text{ kg} \\ &= 22.8\text{ kg} \end{aligned}$$

Since 2.2 has only one decimal place, the final answer using the correct number of significant digits is 22.8 kg.

2. When *multiplying or dividing*, the calculated answer should be rounded to the least number of significant digits of the data values.

Example 6.2

Perform the following multiplication, and express the answer using the correct number of significant digits.

$$\begin{aligned} x &= (3.87\text{ cm})(0.050\text{ cm})(208\text{ cm}) \\ &= 40.248\text{ cm}^3 \\ &= 40\text{ cm}^3 \end{aligned}$$

Since 0.050 has only 2 significant digits, the final answer must be rounded off to 40 cm^3.

3. If you must perform *more than one mathematical operation* to get the final answer, carry all digits through to the final result, without any rounding. The final answer should then be rounded to the same number of significant digits as the quantity in the original data with the fewest number of significant digits.

Example 6.3

Perform the following calculation, and express the answer using the correct number of significant digits.

$$\begin{aligned} x &= (5.2542 \text{ m}) \div (4.56 \text{ s} - 2.31 \text{ s}) \\ &= 5.2542 \text{ m} \div 2.25 \text{ s} \\ &= 2.3352 \text{ m/s} \\ &= 2.34 \text{ m/s} \end{aligned}$$

Since 4.56 s and 2.31 s each have only 3 significant digits, the final answer must be rounded off to 2.34 m/s.

Scientific Notation

Scientists worldwide use the metric system to record observations. Here in Canada, we use the metric system in everyday life as well. Each place value in the metric system is based on units of 10. When a numeral contains a decimal point, the place values to the left of the decimal point increase by an order of 10, and the place values to the right decrease by an order of 10. For example, the numeral 111.11 represents 1 unit of 100, 1 unit of 10, 1 unit of 1, one unit of $\frac{1}{10}$, and 1 unit of $\frac{1}{100}$.

To write very large or very small numbers using the metric system, digits are separated into groups of 3 by spaces (not commas). For example, three million is written as 3 000 000, and $\frac{1\ 234\ 567}{10\ 000\ 000}$ is written as 0.123 456 7.

Note that a zero is always included before the decimal point for numbers less than 1. However, spaces are not used when a number contains only four digits before or after the decimal point (e.g., 6500 or 0.1234). As you can see, writing very large and very small numbers can become cumbersome. Very large or very small numbers are therefore often written using scientific notation.

In scientific notation, numbers are written with only one digit before the decimal point, and then multiplied by a power of 10. The power is a superscript to the base 10, which is called an exponent. For example, the speed of light is 299 792 458 m/s. In scientific notation, this number is written as $2.997\ 924\ 58 \times 10^8$ m/s. Numbers that are less than 1 are written using negative exponents. For example, the mass of a proton is 0.000 000 000 000 000 000 000 017 g; in scientific notation, this number is written as 1.7×10^{-24} g.

Example 6.4

The scientist Avogadro deduced that there are always 602 000 000 000 000 000 000 000 particles in 1 mol of a substance. Express Avogadro's number in scientific notation to 3 significant digits.

In scientific notation, there must be only 1 digit before the decimal place. Therefore, you need to move the decimal place over, and then multiply by 10 to the correct power to give the original number, expressed as an exponent.

Avogadro's number
= 602 000 000 000 000 000 000 000 particles/mol
= $6.02 \times$ 100 000 000 000 000 000 000 000 particles/mol
= 6.02×10^{23} particles/mol

Avogadro's number in scientific notation to 3 significant digits is 6.02×10^{23} particles/mol.

Rules for Calculations Using Scientific Notation

Whenever a quantitative measurement is recorded, it must be written with the correct number of significant digits. Scientific notation allows you to clearly indicate the number of significant digits. When performing a calculation with numbers written in scientific notation, you must use the following mathematical rules for working with exponents. You must also remember to apply the guidelines for determining the correct number of significant digits in the final answer. The exponent of a number in scientific notation does not affect the number of significant digits.

1. To *add or subtract* numbers in scientific notation, the numbers must first be written so that they have the same exponents. The coefficients may then be added like any integer. The exponent in the final answer remains the same.

Example 6.5

Perform the following addition, and express the final answer using the correct number of significant digits.

$$\begin{aligned} x &= (5.4 \times 10^3 \text{ mol}) + (6.82 \times 10^2 \text{ mol}) \\ &= (5.4 \times 10^3 \text{ mol}) + (0.682 \times 10^3 \text{ mol}) \\ &= 6.082 \times 10^3 \text{ mol} \\ &= 6.1 \times 10^3 \text{ mol} \end{aligned}$$

Since this is an addition calculation, the number of significant digits in the final answer is determined by the number in the original data with the least number of decimal places. In this case, this is one decimal place in the number 5.4. The final answer therefore must be rounded to one decimal place, or 6.1×10^3 mol.

2. To *multiply* numbers that are written in scientific notation, the coefficients are multiplied first, and then the exponents are added.

Example 6.6

Perform the following multiplication, and express the final answer using the correct number of significant digits.

$$\begin{aligned} x &= (6.02 \times 10^{23} \text{ atoms}/\cancel{\text{mol}})(3.2 \times 10^{24} \cancel{\text{mol}}) \\ &= (6.02 \times 3.2) \times 10^{23+24} \text{ atoms} \\ &= 19.264 \times 10^{47} \text{ atoms} \\ &= 1.9 \times 10^{48} \text{ atoms} \end{aligned}$$

This is a multiplication, so the number of significant digits in the final answer is determined by the smallest number of significant digits in the original data. In this case, there are 2 significant digits in the number 3.2×10^{24}. The final answer therefore must be rounded to 2 significant digits, or 1.9×10^{48} atoms.

3. To *divide* numbers written in scientific notation, the coefficients are first divided, and then the exponent of the divisor is subtracted from the exponent of the number being divided.

Example 6.7

Perform the following division, and express the final answer using the correct number of significant digits.

$$\begin{aligned} x &= (6.5 \times 10^{7} \text{ g}) \div (3.41 \times 10^{5} \text{ L}) \\ &= (6.5 \div 3.41) \times 10^{7-5} \text{ g/L} \\ &= 1.9061583 \times 10^{2} \text{ g/L} \\ &= 1.9 \times 10^{2} \text{ g/L} \end{aligned}$$

Since this is a division, the number of significant digits in the final answer is determined by the smallest number of significant digits in the original data. In this case, there are 2 significant digits in the number 6.5×10^{7}. The final answer therefore must be rounded to 2 significant digits, or 1.9×10^{2} g/L.

Percents

A percent is an expression of a quantity in terms of hundredths. Percents can be calculated from any ratio of quantities by conversion to an equivalent ratio with a denominator of 100.

$$\text{percent} = \frac{\text{quantity}}{\text{total}} \times 100\%$$

Example 6.8

If you receive a score of 38 correct answers on an examination containing 40 questions, what percent of the questions did you get correct?

The ratio of correct answers to total questions can be written:

$$\frac{38}{40} = \frac{x}{100}$$

Rearranging the equation to solve for x, we get:

$$\begin{aligned} x &= \frac{38 \times 100}{40} \\ &= 95 \end{aligned}$$

Therefore, the percent of questions you got correct is $\frac{95}{100}$, or 95%.

When percents are calculated from measured quantities they must also be expressed using the correct number of significant digits, by applying the rules for calculations with significant digits.

Example 6.9

The incoming radiation to Medicine Hat on November 6, 2002 was reported as follows: 6.3% was reflected back to space, 27.9% was reflected by cloud cover, 8.9% was reflected by Earth's surface, 12.5% was absorbed by greenhouse gases, 4.7% was absorbed by clouds, and 39.7% was absorbed by Earth's surface. What is the total percent of incoming radiation that was absorbed?

Absorbed radiation includes the following: 12.5% absorbed by greenhouse gases, 4.7% absorbed by clouds, and 39.7% absorbed by Earth's surface.

$$\begin{aligned} \text{Percent absorbed} &= 12.5\% + 4.7\% + 39.7\% \\ &= 56.9\% \end{aligned}$$

Since this is an addition, the number of significant digits in the final answer is rounded to the lowest number of decimal places in the original data. Therefore, the total percent of incoming radiation that was absorbed is 56.9%.

Percent Error

In some experiments, the results are compared with a theoretical value, which may be either a value computed for the experiment (such as the theoretical efficiency of an engine) or a standard value (such as the theoretical specific heat capacity of a substance). Percent error communicates the difference between an experimental value and the theoretical value. Percent error is the result of all the following: experimental error (variability between trials of an experiment); readability and precision of instruments; whether all instruments are functioning properly; and whether the experimenter has made any mistakes during the procedure.

Percent error is calculated using the following formula:

$$\text{percent error} = \left| \frac{\text{experimental value} - \text{theoretical value}}{\text{theoretical value}} \right| \times 100\%$$

Example 6.10

A student conducted an experiment to determine the specific heat capacity of iron. The experimental value was 0.51 J/g·°C. The theoretical value is 0.449 J/g·°C. Calculate the percent error.

$$\text{percent error} = \left| \frac{\text{experimental value} - \text{theoretical value}}{\text{theoretical value}} \right| \times 100\%$$

$$= \left| \frac{0.51\ \text{J/g}\cdot{}^\circ\text{C} - 0.449\ \text{J/g}\cdot{}^\circ\text{C}}{0.449\ \text{J/g}\cdot{}^\circ\text{C}} \right| \times 100\%$$

$$= 13.6\%$$

$$= 14\%$$

The percent error of the experimental value for the specific heat capacity of iron is 14%.

Problem-Solving Strategies

Analyzing scientific data often involves mathematical equations. All the variables that describe quantities in mathematical equations have units. Therefore, when working with equations in science, you must do two things: solve for the appropriate variables in the equation, and determine the units of the final answer.

The following steps will help you determine the variable you need to solve for and ensure that the final answer is expressed in the appropriate units.

1. Analyze the problem.
   - Read the entire problem carefully.
   - Identify and list all the given variables in the problem (those for which numerical values are given), including their units.
   - Identify the required variable (the value that needs to be calculated). This variable will be the one variable that is not on your list of given variables.
   - Write down the equation that contains all the variables and that can be used to solve the problem.
2. Use the appropriate conversion factors to make any necessary changes in the units of quantities in the problem, using unit analysis to check that your conversion is correct.
   - Determine the units in which the required variable must be expressed. Check that all the given variables are expressed in units that will allow you to compute the required value in the correct units.
   - If necessary, convert the units of any variable to the appropriate units needed to solve the equation using conversion factors.
3. Solve the equation for the required variable.
   - If necessary, isolate the required variable using formula manipulation. Formula manipulation rearranges the equation into a form that allows you to solve for the required variable.
   - Enter all the given variables in the equation, including their units.
   - Perform the calculations, using unit analysis to cancel out repeated units.
4. Evaluate your answer.
   - Does the final result make sense? If not, recheck your work.
5. Write a statement to answer the question posed in the original problem.

Conversion Factors and Unit Analysis

Suppose you are given a volume in millilitres and need to convert the units of this volume to litres. You can convert between these units by using a conversion factor. A conversion factor is a ratio that relates two quantities expressed in different units when the quantities are of the same attribute (for example, volume). It is the ratio of the required unit to the given unit.

For example, to determine the conversion factor (ratio) of millilitres (the given unit) to litres (the required unit), first determine how many litres are in 1 mL. Recall the relationship between the given unit and the required unit. In this case, 1000 mL are equal to 1 L.

$$1000 \text{ mL} = 1 \text{ L}$$

Then divide both sides of the equation by the appropriate amount to get 1 of the given units on one side of the equation (in this case, 1 mL).

$$\frac{1000 \text{ mL}}{1000 \text{ mL}} = \frac{1 \text{ L}}{1000 \text{ mL}}$$

$$1 \text{ mL} = \frac{1 \text{ L}}{1000 \text{ mL}}$$

Note that multiplying by a conversion factor does not change the value of the measurement, only its unit. Unit analysis (also called dimensional analysis) is a tool for keeping track of units in a calculation, to ensure that the final answer is expressed in the correct units. The following example shows how to use conversion factors and unit analysis to express, in litres, a quantity that was given in millilitres.

Example 6.11

Express the volume 225 mL in litres.

First, write the given variable:
volume, $V = 225$ mL

Now multiply the variable by the conversion factor, and cancel out any repeated units:

$$\text{volume in litres, } V_{(L)} = V \times \frac{1 \text{ L}}{1000 \text{ mL}}$$

$$= 225 \cancel{\text{mL}} \times \frac{1 \text{ L}}{1000 \cancel{\text{mL}}}$$

$$= 0.255 \text{ L}$$

The volume of 225 mL is equal to 0.255 L.

Formula Manipulation

When solving a problem, you may find that the equation is not in the correct form to solve for the variable that you need. You then need to rearrange the equation so that the variable in which you are interested is the only variable on one side of the equation, and the known quantities are on the other side. To isolate a variable, you must always perform the same mathematical operation on both sides of the equation.

Example 6.12

Iron has a specific heat capacity, c, of 0.449 J/g·°C. What was the mass, m, of a piece of iron if the temperature change, Δt, was 10.0°C when 449 kJ of thermal energy, Q, were added? Use the formula $Q = mc\Delta t$.

The given variables are:
$c = 0.449$ J/g·°C,
$\Delta t = +10.0$°C,
$Q = 449$ kJ.
The required variable is mass, m.

Because c is expressed in units of J/g·°C, you must first convert Q from kJ to J. Since 1 kJ = 1000 J, the unit conversion is:

$$Q = 449 \text{ kJ}$$

$$= 449 \cancel{\text{kJ}} \times \frac{1000 \text{ J}}{1 \cancel{\text{kJ}}}$$

$$= 449\,000 \text{ J}$$

Next, rearrange the formula $Q = mc\Delta t$ to solve for m. You can isolate m by dividing both sides of the equation by $c\Delta t$.

$$Q = mc\Delta t$$

$$\frac{Q}{c\Delta t} = \frac{m\cancel{c\Delta t}}{\cancel{c\Delta t}}$$

$$\frac{Q}{c\Delta t} = m$$

Therefore, $m = \dfrac{Q}{c\Delta t}$

You can now substitute the given variables with their units and work out the answer. Remember to include and cancel out units at each step.

$$m = \frac{Q}{c\Delta t}$$

$$= \frac{449\,000 \cancel{\text{J}}}{\left(0.449 \frac{\cancel{\text{J}}}{\text{g}\cdot\cancel{°\text{C}}}\right)(10.0\cancel{°\text{C}})}$$

$$= 100\,000 \text{ g}$$

$$= 1.00 \times 10^5 \text{ g}$$

$$= 100 \text{ kg}$$

The mass of iron was 100 kg.

Student Reference 7: Graphing

Data collected during an experiment is usually recorded in a data table. You can find out more about making a data table in *Student Reference 2: The Inquiry Process*. For quantitative data, analyzing the relationship between the responding and manipulated variables in an experiment can be made easier by using the data to create a graph. A graph can be thought of as a picture, or visual representation, of the data.

Circle Graphs

A circle graph is useful when you want to display data that are part of a whole. There are no manipulated or responding variables in this kind of data. For example, this circle graph (Figure 7.1) shows the percent of different gases in Earth's atmosphere. The graph is given a title that describes the information it contains, and each section of the circle is clearly labelled. Circle graphs may be drawn by hand using paper and pencil, or using technology such as a graphing calculator or spreadsheet software.

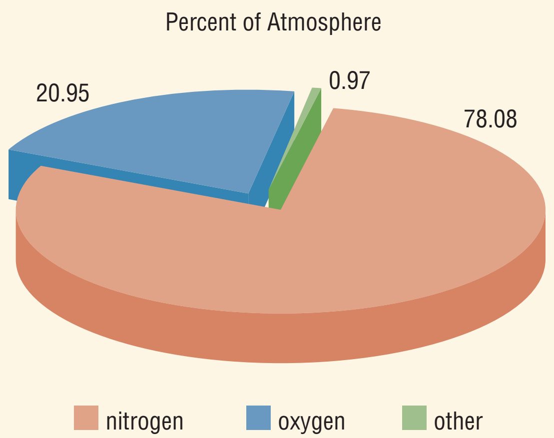


FIGURE 7.1 In this circle graph, the whole circle represents Earth's atmosphere and the parts show the percent of each specific gas.

Bar Graphs

Bar graphs are useful when you want to analyze the relationship between quantitative data in different categories. For example, Table 7.1 shows the average monthly precipitation in Jasper, Alberta. In this example, the manipulated variable is a category, a month, and the responding variable is the average precipitation. The average for each month in the table was calculated independently of the other months. The data for each month are therefore discrete (separate) from the data for all the other months. The researcher would first record the data in a table similar to the one in Table 7.1.

TABLE 7.1 **Average Precipitation per Month in Jasper, Alberta from 1961 to 1990**

Month	Average Precipitation (mm)
Jan	31.1
Feb	17.4
Mar	15.7
Apr	21.2
May	28.6
June	49.9
July	56.2
Aug	50.6
Sept	37.0
Oct	30.9
Nov	28.2
Dec	26.8

Data Source: Environment Canada

On a bar graph, the manipulated variable (e.g., the month) is plotted on the *x*-axis and the responding variable (e.g., the average precipitation) is plotted on the *y*-axis. The *x*-axis is the horizontal axis and the *y*-axis is the vertical axis. The maximum number on the scale of the *y*-axis is determined by the maximum value in the data set. If all the values in the data set are positive, the minimum number on the scale is usually zero. If the data set contains negative numbers, then the minimum value in the data set will be the minimum number on the *y*-axis.

Each category in the data set is drawn as a bar of equal width on the *x*-axis. The height of each bar is determined by the value of the responding variable, and it is drawn according to the scale of the *y*-axis. The graph is given a title, placed at the top of the graph, which describes the information presented. Bar graphs may be drawn by hand using paper and pencil, or using technology such as a graphing calculator or spreadsheet software. As you can see, the changes in the responding variable are a lot easier to see on the graph in Figure 7.2 than in Table 7.1.

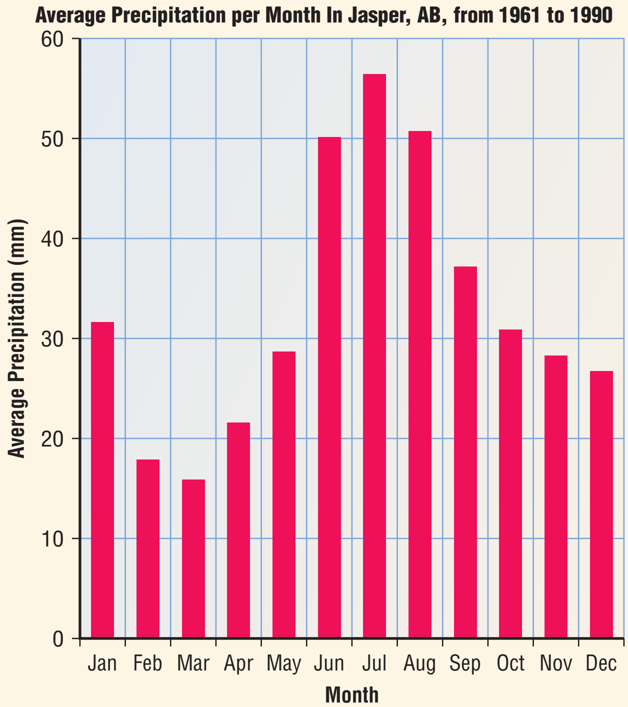


FIGURE 7.2 It is clear that the amount of precipitation is highest from June to August and lowest from February to April from this bar graph.

Scatterplots

Scatterplots are useful for analyzing the relationship between quantitative data in which both the manipulated and responding variables continually change during an experiment. For example, if you were to measure the speed of a bicycle (fitted with a speedometer) rolling down a ramp over time, the manipulated variable (time) and responding variable (speed of the bicycle) would be continuously changing, such as in the data in Table 7.2.

TABLE 7.2 Speed of Bicycle over Time

Time t (s)	Speed v (m/s)
0	0.00
60	0.50
120	1.00
180	1.50
240	2.00
300	2.50

On a scatterplot, the manipulated variable (e.g., time) is plotted on the x-axis and the responding variable (e.g., speed) is plotted on the y-axis. Each axis must be clearly marked with a scale, which must take into account the entire range of measurements to be plotted and use up at least half the size of the graph paper used. The maximum and minimum numbers of the data determine the maximum and minimum numbers on the scales of the axes. It is not necessary for each axis to start at zero, but the axes of a scatterplot usually cross one another at the zero point.

Each piece of data in the table is then plotted by moving over to the correct position on the x-axis and up to the correct position on the y-axis. A point is placed at the intersection of these two positions. If two or more sets of data are plotted on one graph, different colours or shapes are used to plot the different data sets, and a legend is provided to explain the colours or shapes. When the scatterplot is completed, it is given a title that describes the information presented. Scatterplots may be drawn by hand using paper and pencil, or using technology such as a graphing calculator or spreadsheet software. Figure 7.3 shows a scatterplot of the data in Table 7.2.

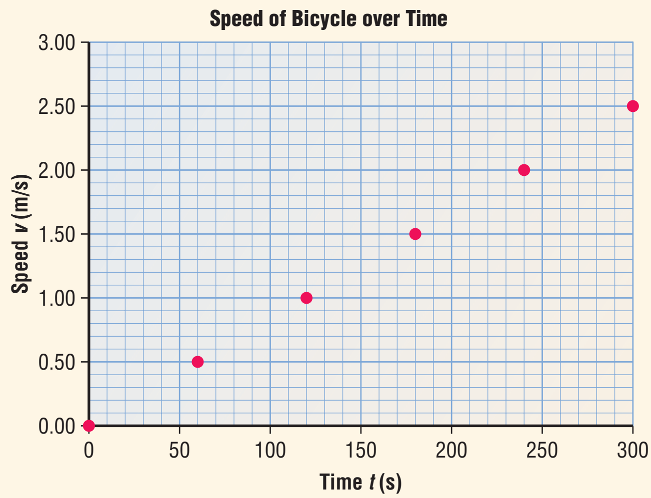


FIGURE 7.3 This scatterplot shows the relationship between the two variables, time and speed, as separate points on the graph.

Drawing the Line of Best Fit

A line of best fit drawn through the plotted data points can help an experimenter determine the relationship between variables more clearly. Scatterplots that include a line of best fit are commonly called line graphs. You can use a graphing calculator or spreadsheet software to draw a line of best fit on a scatterplot. Note that you cannot create a line of best fit by connecting or trying to draw a curve or straight line through all the data points. Figure 7.4 shows the line of best fit for the scatterplot in Figure 7.3.

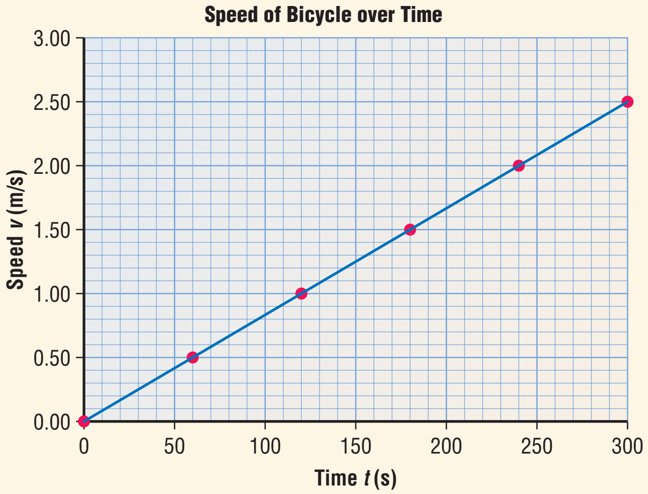


FIGURE 7.4 The line of best fit for these data goes through all the data points.

If a point is far off the line of best fit, such as is shown in Figure 7.5, it indicates that a serious error may have been made. If this occurs, measure the data for that point again. If the same result is obtained, a factor other than those under investigation may be the cause of the variation. For example, suppose another student's data table contained the entries in Table 7.3.

TABLE 7.3 Speed of Bicycle over Time

Time t (s)	Speed v (m/s)
0	0.00
60	0.50
120	0.76
180	1.83
240	2.00
300	2.50

The line of best fit for the scatterplot of these data is shown in Figure 7.5.

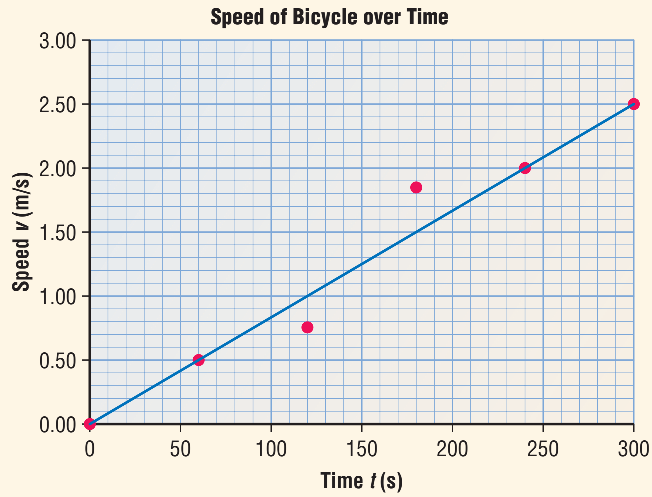
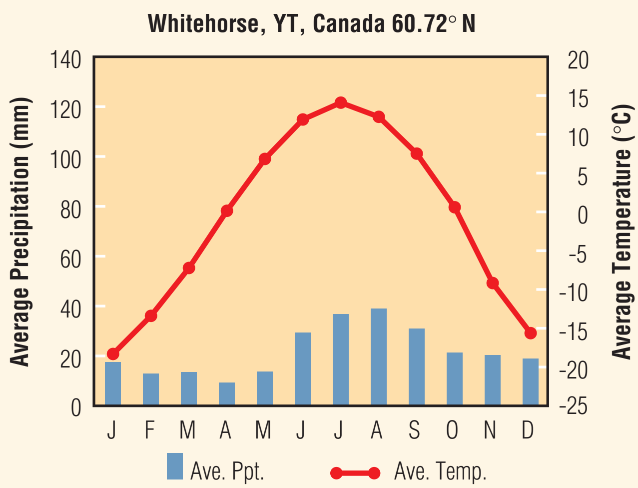


FIGURE 7.5 Although the line of best fit is still a straight line, it does not pass through all the data points. The points that are not on the line should be measured again, to ensure they are accurate.

Combining Different Types of Graphs

In some cases, two different types of data may be combined on one graph. For example, in Unit D you will create climatographs, which are graphical representations of the average climate of an area. A climatograph is composed of a bar graph showing average precipitation per month, and a line graph showing average temperature per month. There are two vertical axes, as shown in Figure 7.6. The vertical axis on the left presents the scale for the precipitation data, and the vertical axis on the right presents the scale for the temperature data.



Source: Environment Canada

FIGURE 7.6 A climatograph combines a line graph with a bar graph.

Interpolation and Extrapolation

Graphing data can also allow us to estimate values for data points that we did not or cannot directly measure. Interpolation is the process of estimating a value that is between two directly measured data points of a variable. Data can often be interpolated using a line graph. For example, in Figure 7.4, you might want to interpolate the speed of the bicycle after 250 s. Figure 7.7 shows the procedure for interpolating data from a linear relationship on a line graph.

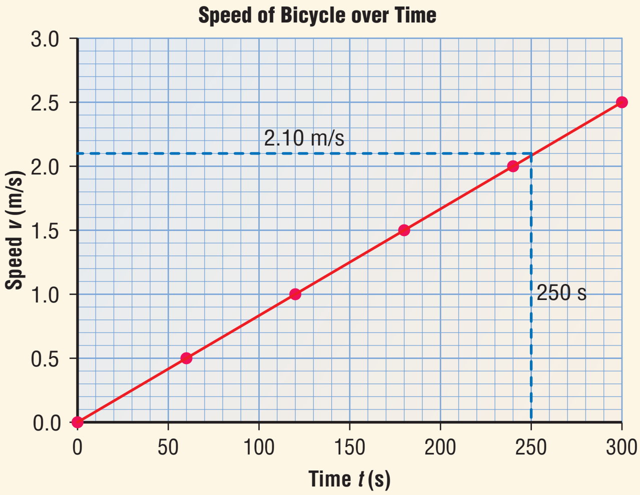


FIGURE 7.7 Values can be interpolated from a line graph when the relationship between the variables is linear.

First, locate the data point in which you are interested on the appropriate axis. In this example, this is the 250 s point on the *x*-axis. Next, draw a perpendicular line from this point to the line of best fit. At the point where the perpendicular line and the line of best fit intercept, draw a second line at a right angle to the first line and all the way to the second axis. In this example, this is the line from the intersection point to the *y*-axis. You can now read the interpolated value for the second variable using the scale of this axis (2.10 m/s in this example).

There is always some inaccuracy involved in interpolation, because we assume that the trend of the line continues between measured points. This assumption may not always be valid. There is also inaccuracy due to measurement, especially if you are working with pencil and paper.

Extrapolation is the process of estimating the values of a data point beyond the limits of the known or measured values. However, there is a considerable risk of inaccuracy, because we assume that the trend of the curve or straight line continues outside the range of the data. A dotted line is usually used to show extrapolation of a line. Figure 7.8 shows the extrapolated value for the speed of the bicycle at 400 s.

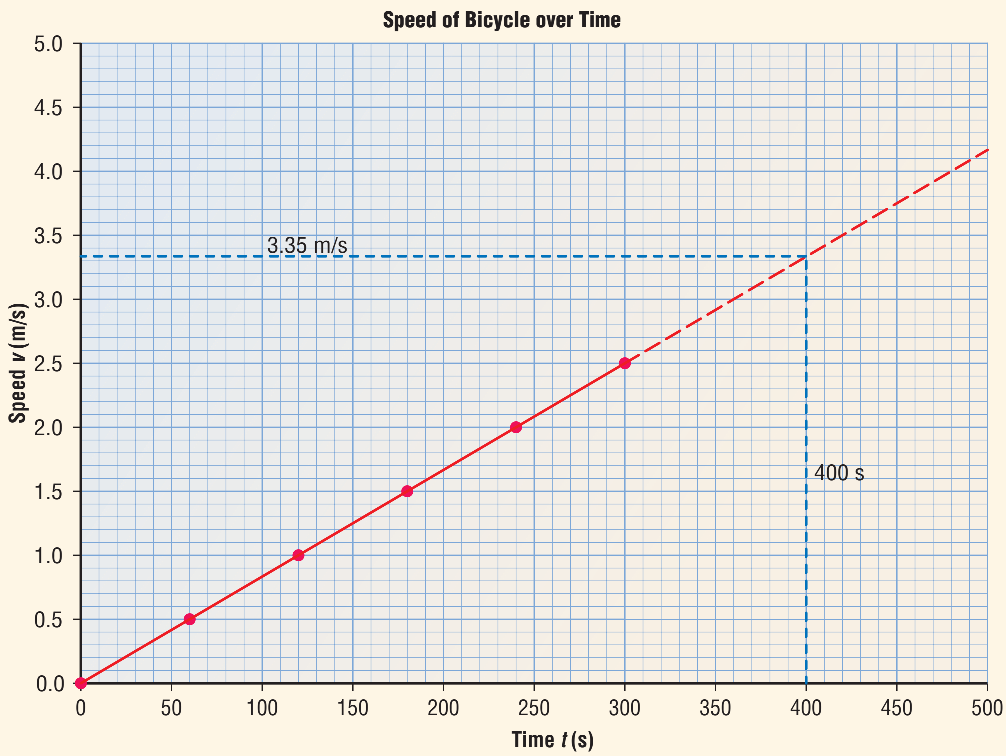


FIGURE 7.8 To extrapolate the speed of the bicycle at 400 s, extend the straight line of the graph beyond the last data point. The extrapolated line is shown as a dotted red line. The speed of the bicycle at 400 s can then be determined by drawing a straight line (horizontal blue dotted line) to the *y*-axis from the intersection of the 400 s point on the *x*-axis with the extrapolated line (vertical blue dotted line). This gives an extrapolated speed of 3.35 m/s.

Relationships between Variables

The main function of graphing is to help us to understand the relationships between variables. The previous examples have all shown linear relationships between the manipulated and responding variables. That is, a straight line defines the relationship between the variables. Line graphs (scatterplots with a line of best fit) can also tell you if a relationship between variables is non-linear. For example, consider the following data on the speed of two different bicycles as they passed markers at different distances during a race (Table 7.4).

TABLE 7.4 Speed of Two Bicycles Passing Markers

Distance d (m)	Speed of Bicycle A v_A (m/s)	Speed of Bicycle B v_B (m/s)
0.0	0.0	0.0
1.0	0.5	0.5
2.0	1.0	1.0
3.0	1.5	2.0
4.0	2.0	3.5
5.0	2.5	5.5

The line graph for these data is shown in Figure 7.9. The plotted data for bicycle A show a linear relationship between speed and distance. In other words, the cyclist is increasing speed, or accelerating, at a constant rate. The plotted data for bicycle B, however, show a non-linear relationship between speed and data. In this case, the cyclist is accelerating at a greater rate as the race continues.

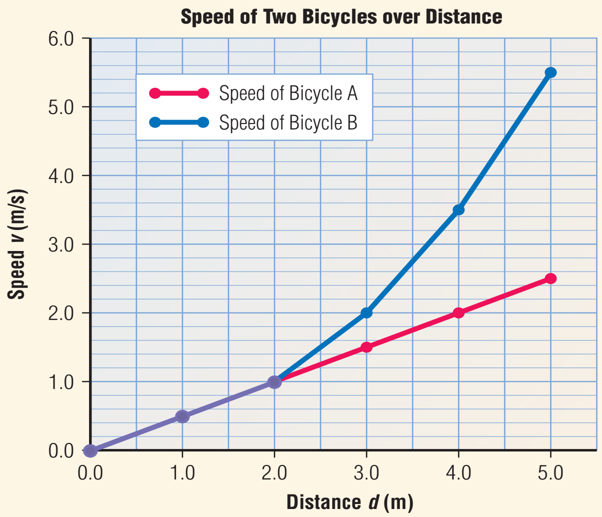


FIGURE 7.9 The data for bicycle A show a linear relationship between speed and distance, but the data for bicycle B show a non-linear relationship between these two variables.

Calculating the Slope

If the relationship between two variables is linear (i.e., the line of best fit is straight), we can use the graph to find the slope of the line. The slope of the line is defined as the ratio of the rise to the run. A run is a horizontal line drawn below the curve of the graph, touching the curve at one end. A rise is a vertical line joining the free end of the run to the curve.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

When two variables have a linear relationship, then the manipulated variable and the responding variable increase or decrease in proportion to each other. When one variable varies by one unit, the other will always vary by a certain number of units. The slope of the graph gives us this number.

Figure 7.10 shows how to calculate the slope of the graph first shown in Figure 7.4. To find the slope, draw any convenient run on the graph. Your calculations will be simpler and more accurate if you make the run a large whole number. Then draw the corresponding rise.

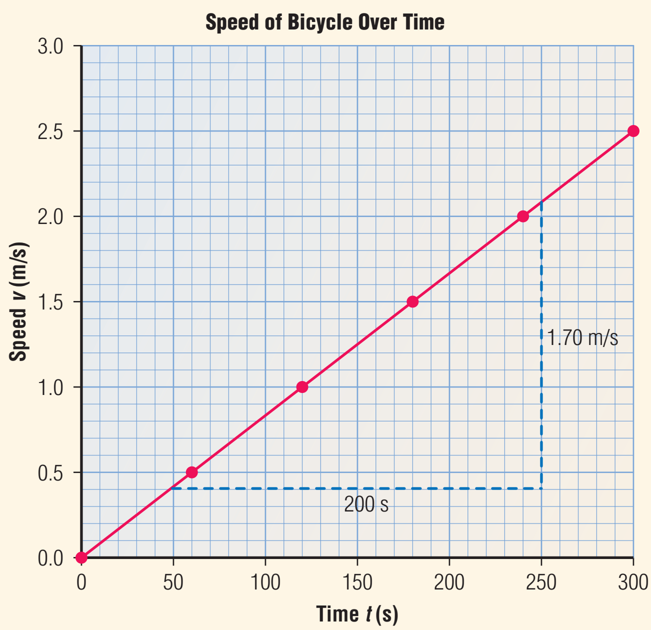


FIGURE 7.10 This graph has a run of 250 s – 50 s = 200 s and a rise of 2.10 m/s – 0.40 m/s = 1.70 m/s.

To calculate the slope for this graph, carry out the following steps:

$$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}}\\ &= \frac{y_2 - y_1}{x_2 - x_1}\\ &= \frac{2.10\,\frac{\text{m}}{\text{s}} - 0.40\,\frac{\text{m}}{\text{s}}}{250\text{ s} - 50\text{ s}}\\ &= \frac{1.70\,\frac{\text{m}}{\text{s}}}{200\text{ s}}\\ &= 0.0085\,\frac{\text{m}}{\text{s}^2}\\ &= 8.5 \times 10^{-3}\,\frac{\text{m}}{\text{s}^2}\end{aligned}$$

The slope of the graph is 8.5×10^{-3} m/s^2

We can therefore say a lot of things about the relationship between speed and time from this graph. We can say that speed and time have a linear relationship, i.e., the two variables are directly proportional to each other. From calculating the slope of the graph, we know that the proportion by which the speed of the bicycle increases is 8.5×10^{-3} m/s for every 1 s increase in time.

Notice that the slope has units. Examining the units will help you determine the meaning of the slope. The slope of a speed versus time graph such as Figure 7.10 is in units of m/s^2, which is the unit for acceleration. By finding the slope of the line, we have found the value for a quantity that we did not directly measure, in this case, the acceleration of the bicycle.

Calculating the Area under a Line

The area under the line of a graph of a linear relationship can also be used to find the value of a variable that was not directly measured. The relationship between variables is often described by an equation. For example, the speed (v) of any object can be calculated from the following formula:

$$v = \frac{\Delta d}{\Delta t}$$

where Δd is the change in distance and Δt is the change in time.

By formula manipulation, this formula can be rearranged to solve for Δd. That is

$$\Delta d = v\Delta t$$

The data in Table 7.2 do not include any measurement for the distance the bicycle travelled. We could use this formula to determine Δd for any time point, or we can determine Δd from the area under the line.

Note that the shaded area under the line in Figure 7.11 forms a triangle. The area under the line can therefore be determined as follows:

$$\begin{aligned}\text{area under the line} &= \text{area of a triangle}\\ &= \frac{1}{2}\,\text{base} \times \text{height}\\ \text{area} &= \frac{1}{2}\,\Delta t \times v\\ &= \frac{1}{2}\,200\,\cancel{\text{s}} \times 1.65\,\frac{\text{m}}{\cancel{\text{s}}}\\ &= 165\text{ m}\end{aligned}$$

The distance that the bicycle travelled in 200 s was 165 m.

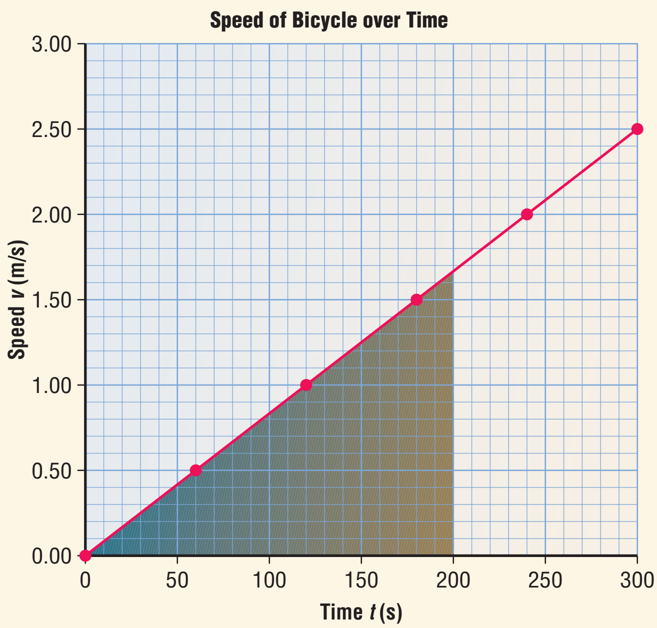


FIGURE 7.11 The area under this line is a triangle. You therefore can use the formula for the area of a triangle to find the area.

Student Reference 8: The Compound Light Microscope

A microscope allows us to see an image of an object that is too small to see with the unaided human eye. A light microscope functions by focussing a beam of light through the object into the lens of the microscope. A compound light microscope is any light microscope that contains more than one lens. The compound light microscope you will use in the science classroom contains an eyepiece lens and a number of objective lenses. Each objective lens is a combination of two lenses made of different kinds of glass.

The Parts of the Microscope

It is important to know the location and function of the parts of the microscope in order to use it correctly. These are shown in Figure 8.1.

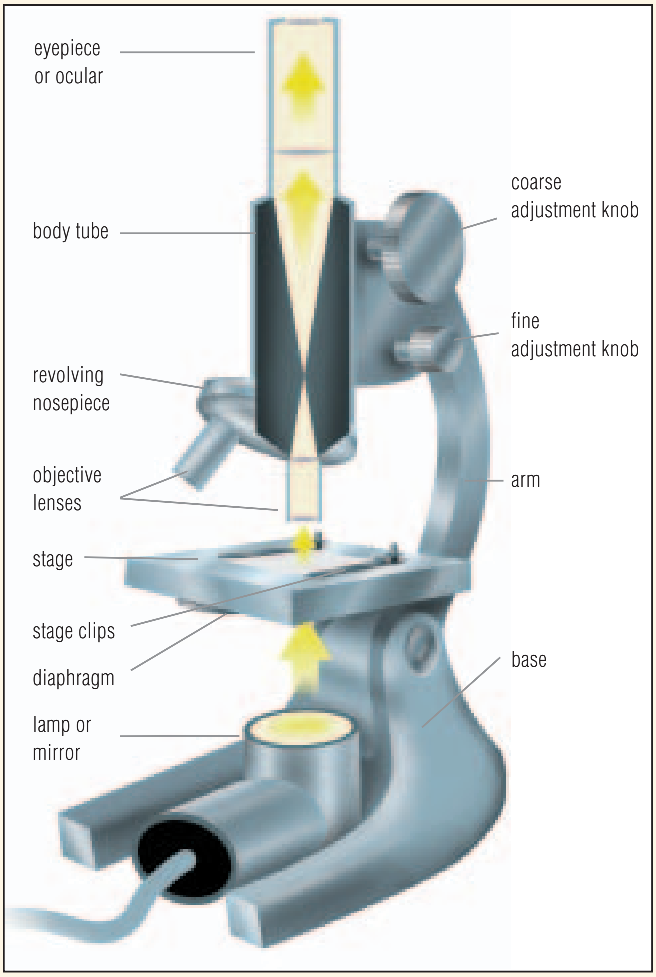


FIGURE 8.1 A compound light microscope

Using the Microscope

1. Carry the microscope with two hands, grasping the arm of the microscope with one hand and holding the base of the microscope with the other. Place the microscope on the table or bench so that the arm is facing you.
2. Plug in the microscope and turn on the light.
3. Rotate the nosepiece until the objective lens with the lowest power is in place.
4. Place a microscope slide on the stage and secure with the stage clips.
5. Watch the stage from one side of the microscope and slowly lower the nosepiece with the coarse adjustment until it is as low as possible. Ensure the lens does not touch the slide.
6. Look through the eyepiece. Slowly turn the coarse adjustment so that you move the lens away from the slide. Stop when the image comes into view.
7. Use the fine adjustment to sharpen the focus of the image.
8. If you need to view the object under higher magnification, watch from the side of the microscope and rotate the nosepiece until the next higher power objective lens is in place. Ensure the lens does not touch the slide. Use only the fine adjustment knob to focus the image.

Magnification and Field of View

Each lens on the compound microscope will magnify a sample to a different degree. Magnification is calculated by multiplying the power of the ocular lens (usually 10× power) by the magnification of the objective lens you are using.

magnification = (power of ocular lens)(power of objective lens)

For example, if you are viewing a slide using a 4× power objective lens, the magnification of the image would be (10×)(4×) = 40×.

The field of view is the entire area that you see when you look through the microscope. The diameter of the field of view varies with the particular objective lens you are using. The diameters of the field of view for low-power (4×) and medium-power (10×) objective lenses can be determined by the following steps:

1. Rotate the objective lens into position.
2. Place a small, transparent, metric ruler on the stage so that it covers about half the stage. The ruler must be small enough to fit on the stage.
3. Using the coarse adjustment knob, bring the ruler into focus. Adjust the placement of the ruler so that the scale crosses the centre of the circle (the diameter), as shown in Figure 8.2.
4. Use the fine adjustment knob to get a clear, sharp image. If necessary, adjust the ruler so that one of the markings on the left side is exactly at the edge of the diameter.

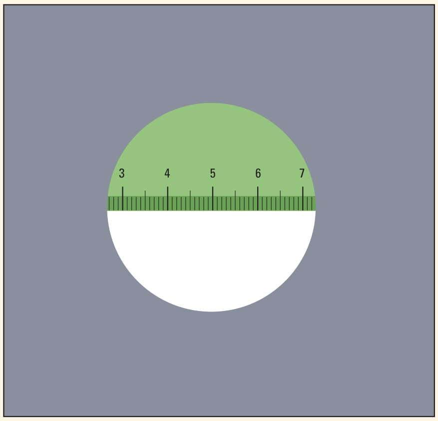


FIGURE 8.2 Move the ruler so that you are measuring the diameter (width) of the field of view from left to right.

5. Determine the diameter of the field of view in millimetres, using the scale on the ruler. Convert the millimetre reading to micrometres. This is the field of view for the magnification used.

You cannot measure the diameter of the field of view of a high-power (40×) objective lens using this method, because the field of view is less than 1 mm. However, you can estimate the diameter of the field of view of a high-power objective lens by using ratios. As you increase magnification by a certain amount, you decrease the diameter of the field of view by the inverse of that amount.

Therefore, you can determine the diameter of the field of view of a high-power (HP) objective lens by using the following ratio:

$$\frac{\text{HP field diameter}}{\text{LP field diameter}} = \frac{\text{LP magnification}}{\text{HP magnification}}$$

Example 8.1

A student measured the field diameter of a microscope using the 4× and 10× objective lenses.

Objective Lens	Magnification of Objective Lens	Field Diameter (mm)	Field Diameter (μm)
low power	4×	4.5	4500 or 4.5×10^3
medium power	10×	1.1	1100 or 1.1×10^3

Calculate the field diameter of a high-power (40×) objective lens.

$$\frac{\text{HP field diameter}}{\text{LP field diameter}} = \frac{\text{LP magnification}}{\text{HP magnification}}$$

$$\begin{aligned}\text{HP field diameter} &= \text{LP field diameter} \times \frac{\text{LP magnification}}{\text{HP magnification}} \\ &= 4500\ \mu\text{m} \times \frac{(4\cancel{\times})}{(40\cancel{\times})} \\ &= 450\ \mu\text{m} \\ &= 4.5 \times 10^2\ \mu\text{m}\end{aligned}$$

The field diameter of the high-power (40×) objective lens is 4.5×10^2 μm.

Note that when the magnification increases by a factor of 10, such as from 4× to 40×, the field diameter decreases by the same factor (10×), from 4500 μm to 450 μm.

Once you have estimated the diameter of the field of view of an objective lens, you can estimate the size of any structure you are viewing with that lens. Compare the size of the structure with the diameter of the field of view. For example, if a cell component takes up one-tenth of a field of view that has a diameter of 400 μm, then the cell component is about one-tenth of 400 μm, or 40 μm.

Preparing a Wet Mount

1. Obtain a clean microscope slide and coverslip. In a wet mount, the coverslip serves three functions: it flattens the sample, it prevents the sample from drying out, and it protects the objective lens from contamination.
2. Place your sample in the centre of the slide. The specimen must be thin enough for light to pass through.
3. With an eyedropper, place a drop of water on the sample, as shown in Figure 8.3.

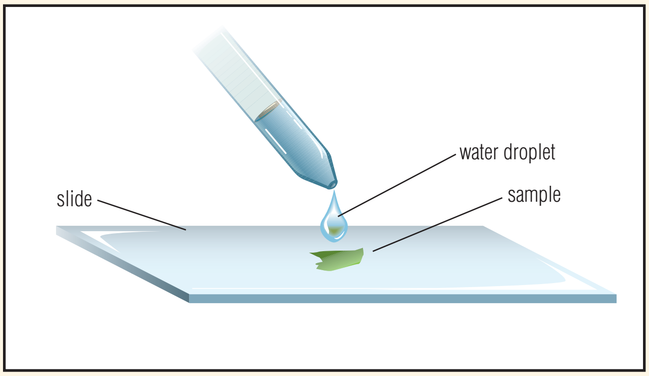


FIGURE 8.3 Step 3

4. Place the coverslip at an angle at one end of the drop of water (Figure 8.4(a)). Carefully lower the coverslip to cover the sample, being careful not to trap any air. It may be helpful to use a probe or toothpick to lower the coverslip.
5. If you do get air bubbles, gently tap the slide with a probe to release them (Figure 8.4(b)).

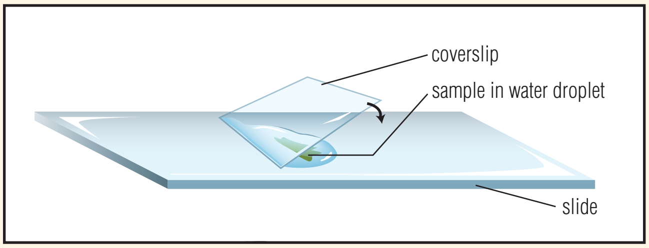
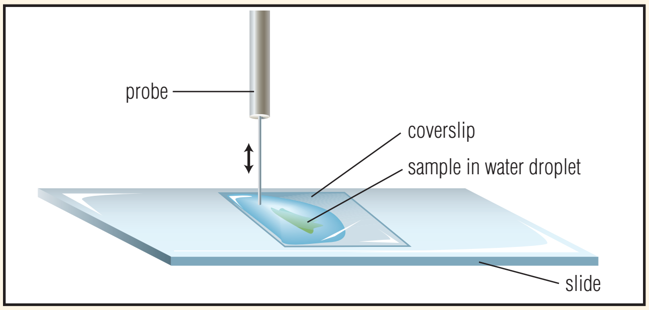


FIGURE 8.4 (a) Lower the coverslip slowly to avoid trapping air.



(b) Tapping gently on the coverslip can release any air bubbles that do occur.

Staining Samples

The parts of a cell are composed of various substances, and the different cell components react differently to many chemicals. Stains are chemicals that react in specific ways to different cell components. Stains therefore make it easier to distinguish the components of a cell. Some stains will dye only certain parts of the cell. Others change colour depending on the substances that comprise the different cell components.

There are many ways to stain cells, but one of the most common is the flow technique. This technique may be used to stain cells with, for example, iodine or methylene blue. The flow technique consists of the following steps:

1. Prepare a wet mount slide, as described at left.
2. Place a drop of stain at the edge of one side of the coverslip.
3. Obtain a small piece of paper towel or tissue paper. Place the paper against the edge of the coverslip on the side opposite to the stain, as shown in Figure 8.5(a).
4. Allow the paper to wick the fluid from under the coverslip and draw the stain into the sample, as shown in Figure 8.5(b).
5. Remove the paper when the stain has travelled to the other side of the coverslip.
6. If the stain is too dark, it may be diluted by repeating steps 2 to 5 with a drop of water.

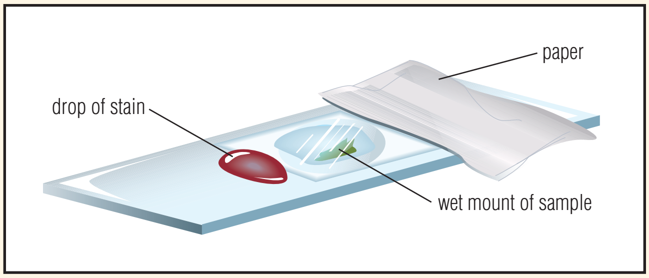
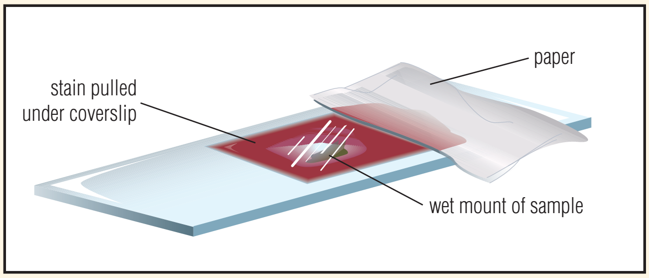


FIGURE 8.5 (a) Place the paper next to the coverslip, being careful not to disturb your sample.



(b) Do not remove the paper until the stain is spread evenly under the coverslip.

Drawing Scientific Diagrams

To record what you observe under a microscope, you will often draw a scientific diagram. Scientific diagrams can also be used to record observations not made with a microscope. For example, they may also be used to show how equipment is set up for an experiment or to record objects observed with the unaided eye. A scientific diagram is different from other types of drawings in that it represents "the real thing." In other words, it is a record of exactly what was observed, with all features accurately drawn and identified.

Guidelines for Drawing Scientific Diagrams

1. Give a title for your diagram at the top of the page. The title should include information about the object shown (Figure 8.6).
2. Use pencil. Do not colour diagrams. Shade areas if necessary.
3. Draw only one diagram on a page. (There are sometimes exceptions to this rule, for example, if you were drawing only very simple diagrams.)
4. Label the parts or structures of the object on the diagram. Use a ruler to draw lines to connect the label to the part or structure.
5. Record the scale of the drawing at the side of the diagram.

Cross Section of Plant Stem

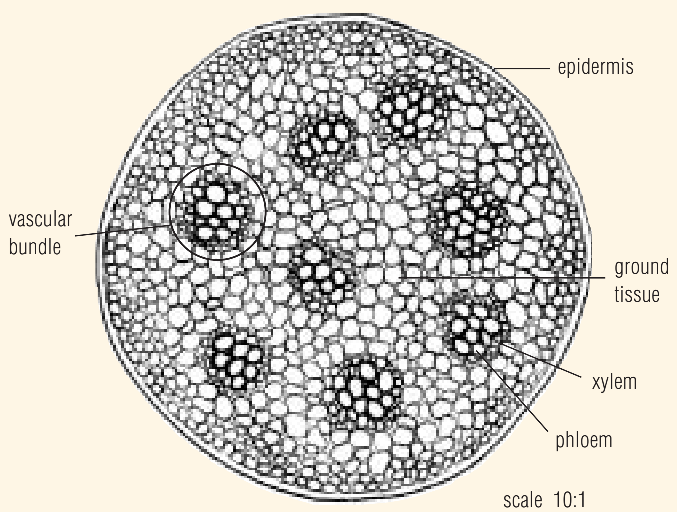


FIGURE 8.6 This example of a scientific diagram shows the features of a cross-sectional view of a plant stem.

When samples have been dissected (cut apart), it is important to note how they were prepared in the title of the diagram. A sample can be prepared as a cross section (across the width), as in Figure 8.6, or as a longitudinal section (lengthwise), as in Figure 8.7.

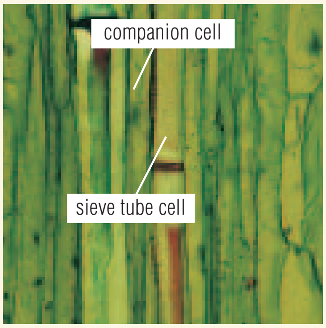


FIGURE 8.7 This longitudinal view of a plant stem shows different features of the plant stem from those shown in a cross section.

Diagrams may be drawn larger, smaller, or the same size as the actual object. The scale of a diagram is the difference between the size of the diagram and the size of the actual object. Scale is often expressed as a ratio, such as in the examples in Table 8.1.

TABLE 8.1 **Actual Size, Diagram Size, and Scale**

Actual Size	Diagram Size	Scale
1.1 mm	11 cm (110 mm)	100:1 (or $\times$ 100)
2.6 m	2.6 cm	1:100 (or $\times$ 0.01)

When using a microscope, the actual size of the object is usually estimated by comparing it to the diameter of the field of view.

To calculate actual size and scale for a scientific diagram, you must first measure the field diameter (if you are using a microscope) and the size of the finished diagram. Actual size and scale can then be calculated using the following relationships:

$$\text{actual size of object} = \frac{\text{field diameter}}{\text{number of objects estimated to fit across field of view}}$$

$$\text{scale} = \frac{\text{diagram size of objects (units)}}{\text{actual size (units)}}$$